

Two-loop QCD corrections to the vector form factors for the heavy-quark photo-production

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We review on the calculation of the heavy-quark photo-production vector form factors, with the full dependence on the mass of the heavy-quark. The Feynman diagrams are evaluated within the dimensional regularization scheme and expressed in Laurent series of $\epsilon = (4 - D)/2$, where D is the space-time dimension. The coefficients of the expansion are expressed in terms of harmonic polylogarithms. The numerical evaluation of the analytical formulas and the threshold limit of our results are presented.

The measurement of the forward-backward asymmetry of the production of heavy quarks in $e^+e^- \rightarrow Q\bar{Q}$ processes, such as $A_{FB}^{c,b}$ on the Z^0 peak [1], and, at the next generation of Linear Colliders, also the measurement of A_{FB}^t [2], is a stringent test of the Standard Model, providing a precise determination of $\sin^2 \theta_W^{eff}$ [3].

The present theoretical description includes the NNLO QCD corrections. For massless quarks, they were calculated numerically in [4] and analytically in [5], while in [6] the order $\mathcal{O}(\alpha_S^2)$ -corrections were calculated numerically for the b-quark retaining terms that do not vanish in the small-mass limit (constants and log-enhanced terms), but neglecting both terms containing linear mass corrections, like m_b^2/s , and terms in which such a ratio is enhanced by a power of $\log(s/m_b^2)$. In order to take into account also this kind of terms, a full analytic calculation in which the mass of the heavy quark is kept systematically different from zero is required. Three classes of contributions are involved: the tree level matrix elements for the decay of a vector boson into four partons, at least two of which being the heavy quark-antiquark pair; the one-loop corrected matrix elements for the decay of a vector boson into a heavy quark-antiquark pair plus a gluon; the two-loop corrections to the decay of a vector boson into a heavy quark-antiquark pair.

In this paper we consider the virtual NNLO QCD corrections to the process $\gamma^* \rightarrow Q(p_1)\bar{Q}(p_2)$, with $P = p_1 + p_2$ the momentum of the virtual photon and p_1, p_2 the momenta of the outgoing on-shell quark and antiquark respectively ($p_1^2 = p_2^2 = -m^2$ with m the mass of the heavy quark). The vertex amplitude $V^\mu(p_1, p_2)$ can be expressed in terms of two form factors as:

$$V^\mu(p_1, p_2) = \bar{u}(p_1)\Gamma^\mu(p_1, p_2)v(p_2), \quad (1)$$

$$\Gamma^\mu(p_1, p_2) = F_1(p^2)\gamma^\mu + \frac{1}{2m}F_2(p^2)\sigma^{\mu\nu}P_\nu, \quad (2)$$

where $\bar{u}(p_1), v(p_2)$ are the spinor wave functions of the quark and antiquark, $\sigma^{\mu\nu} = -\frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and $p^2 = P^2/m^2 = -s/m^2$, with s the squared c.m. energy.

At the two-loop level, the contributions to $F_1(p^2)$ and $F_2(p^2)$ come from the diagrams shown in Fig. 1. They are evaluated in [7] using a method based essentially on two steps:

1. By means of suitable projector operators, the contributions to $F_1(p^2)$ and $F_2(p^2)$ are expressed in terms of a big number of scalar integrals, whose ultraviolet (UV) and infrared (IR) singularities are regularized within the dimensional regularization scheme. Applying the so-called Laporta algorithm, introduced in [8] and become, in the last few years, a standard technique for the calculation of Feynman diagrams, all these scalar integrals are expressed in terms

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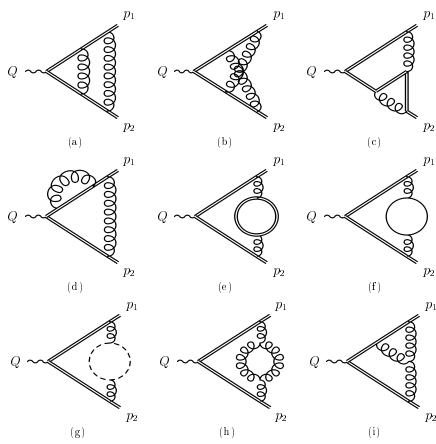


Figure 1. Two-loop Feynman diagrams. The curly lines represent gluons; the double straight lines, quarks of mass m ; the single straight lines, massless quarks and the dashed lines ghosts.

of 34 independent integrals called Master Integrals (MIs). The algorithm is based on Integration by Parts Identities [9], Lorenz invariance identities [10] and General Symmetry Relations. It is completely algebraic and it is implemented in a code written in FORM [11].

2. The calculation of the Master Integrals, then, is carried out [12,13] by means of the Differential Equations technique [14].

The renormalization of UV divergences is performed in a hybrid scheme: we renormalize the mass and wave function of the heavy quark in the on-shell renormalization scheme, while the coupling constant, as well as the light-fermion and gluon wave functions, are renormalized in the $\overline{\text{MS}}$ scheme.

The final result of the calculation is an analytical expression for the form factors, written in terms of harmonic polylogarithms of one variable [15,16]. The result has still IR divergences that appear as poles in $1/\epsilon$. These divergences have to be canceled against those arising from the real radiation, which are not taken into account here.

We present, in this short review, the numerical

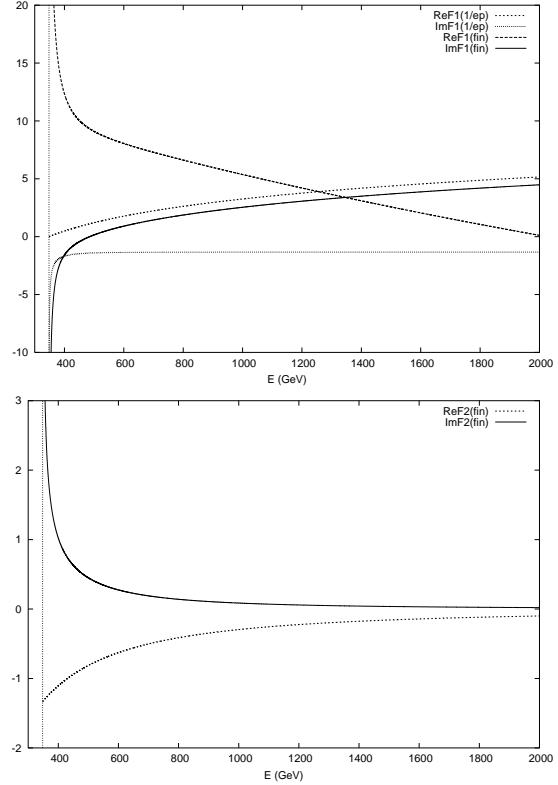


Figure 2. One-loop form factors, F_1 and F_2 , in the case of top-antitop production. The charge form factor (upper plot) has an IR pole, while the magnetic form factor is IR finite. Note the Coulomb divergences of the form factors in the threshold limit. The real part of the pole of F_1 and the real part of F_2 are, instead, finite (0 and $-C_F$ respectively).

evaluation of the different parts of the form factors (real and imaginary parts of the poles and finite parts) at one- and two-loop level above the threshold for the case of the production of a pair of top-antitop quarks (one massive and five light flavours). The numerical evaluation is performed in F77 using the subroutine of [16]. The color charges are set according to the $SU(3)$ group. The c.m. energy runs from the threshold value ($\sqrt{s} \sim 2m$, $m = 178 \text{ GeV}$) to 2 TeV. The plots are shown in Figs. 2, 3 and 4.

The knowledge of the exact analytical formula for the form factors allows a perfect control on

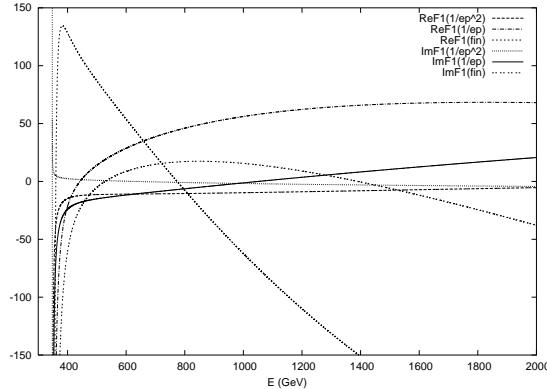


Figure 3. Two-loop charge form factor, F_1 , in the case of top-antitop production. The charge form factor has a double IR pole.

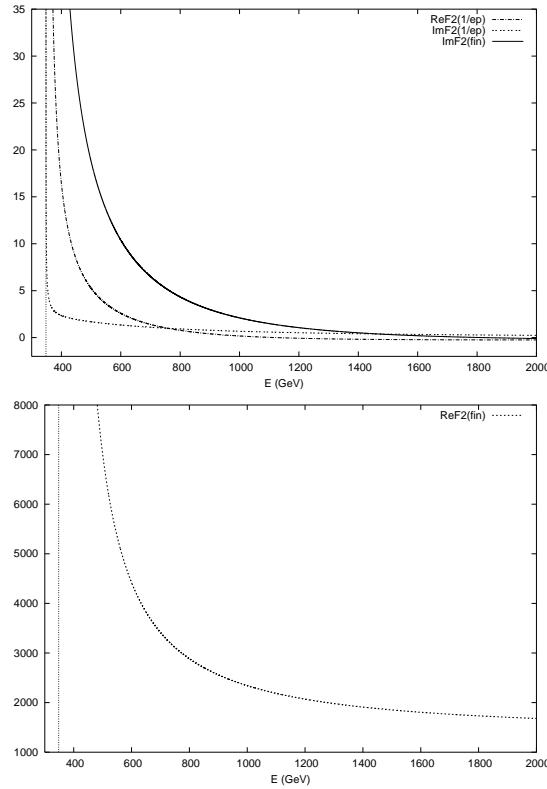


Figure 4. Two-loop magnetic form factor F_2 , in the case of top-antitop production. The magnetic form factor has a simple IR pole.

their numerical evaluation as well as on their analytical behaviour in all the regions of the spectrum. As an example, consider the threshold region. The NNLO determination of the threshold limit of the vector form factors were already present in the literature [17] as an independent calculation, performed with a different technique. With the method outlined above, however, this result can be achieved as a limiting case of the full calculation.

In fact, let us define the real and imaginary parts of the UV-renormalized form factors at one and two loops as:

$$F_i^{(1l,2l)} = C(\epsilon) \left\{ \Re \mathcal{F}_i^{(1l,2l)} + i\pi \Im \mathcal{F}_i^{(1l,2l)} \right\}, \quad (3)$$

with $i = 1, 2$ and $C(\epsilon) = (4\pi)^\epsilon \Gamma(1 + \epsilon)$. Moreover, consider the total cross-section for the process $e^+e^- \rightarrow Q\bar{Q}$ near the threshold:

$$\sigma_{e^+e^- \rightarrow Q\bar{Q}} = \sigma^{(0)} \left[1 + \frac{\alpha_S}{2\pi} \Delta^{(1)} + \left(\frac{\alpha_S}{2\pi} \right)^2 \Delta^{(2)} \right], \quad (4)$$

where $\sigma^{(0)} = (2\pi\alpha^2 e_Q^2/s) \beta(3 - \beta^2)$ is the tree-level cross-section and $\beta = \sqrt{1 - \frac{4m^2}{s}}$ is the quark velocity. The one- and two-loop corrections, $\Delta^{(1)}$ and $\Delta^{(2)}$, can be written in terms of the form factors, up to $\mathcal{O}(\beta^2)$, in the following way:

$$\Delta^{(1)} = 2 \left[\Re \mathcal{F}_1^{(1l)} + \Re \mathcal{F}_2^{(1l)} \right], \quad (5)$$

$$\begin{aligned} \Delta^{(2)} = & \left[\Re \mathcal{F}_1^{(1l)} \right]^2 + 2 \Re \mathcal{F}_1^{(1l)} \Re \mathcal{F}_2^{(1l)} + 2 \Re \mathcal{F}_1^{(2l)} \\ & + \pi^2 \left[\Im \mathcal{F}_1^{(1l)} \right]^2 + 2\pi^2 \Im \mathcal{F}_1^{(1l)} \Im \mathcal{F}_2^{(1l)} \\ & + \left[\Re \mathcal{F}_2^{(1l)} \right]^2 + 2 \Re \mathcal{F}_2^{(2l)} + \pi^2 \left[\Im \mathcal{F}_2^{(1l)} \right]^2. \end{aligned} \quad (6)$$

Using the analytical expressions for the threshold limits of $\Re \mathcal{F}_1^{(1l)}$, $\Im \mathcal{F}_1^{(1l)}$, $\Re \mathcal{F}_2^{(1l)}$, $\Im \mathcal{F}_2^{(1l)}$, $\Re \mathcal{F}_1^{(2l)}$, and $\Re \mathcal{F}_2^{(2l)}$, given in [7], the functions $\Delta^{(1)}$ and $\Delta^{(2)}$ read:

$$\Delta^{(1)} = C_F \left[\frac{6\zeta_2}{\beta} - 8 \right], \quad (7)$$

$$\begin{aligned} \Delta^{(2)} = & C_F^2 \left\{ \frac{12\zeta_2^2}{\beta^2} - \frac{48\zeta_2}{\beta} + 39 - \frac{142}{3}\zeta_2 - 4\zeta_3 \right. \\ & \left. + 24\zeta_2^2 + 32\zeta_2 \ln 2 - 16\zeta_2 \ln \beta \right\} \end{aligned}$$

$$\begin{aligned}
& + C_F C_A \left\{ \frac{1}{\beta} \left[\frac{31}{3} \zeta_2 - 22 \zeta_2 \ln(2\beta) \right] - \frac{151}{9} \right. \\
& \left. + \frac{179}{3} \zeta_2 - 26 \zeta_2 - 64 \zeta_2 \ln 2 - 24 \zeta_2 \ln \beta \right\} \\
& - C_F T_R N_f \left\{ \frac{1}{\beta} \left[\frac{20}{3} \zeta_2 - 8 \zeta_2 \ln(2\beta) \right] - \frac{44}{9} \right\} \\
& + C_F T_R \left\{ \frac{176}{9} - \frac{32}{3} \zeta_2 \right\}, \tag{8}
\end{aligned}$$

where ζ_2 and ζ_3 are the Riemann ζ function evaluated in 2 and 3 respectively. Eqs. (7,8) are in complete agreement with the results in [17].

To conclude, we presented the calculation of the two-loop QCD contributions for the vector form factors of the photo-production of heavy quarks. By means of the Laporta and differential equations methods we were able to calculate analytically the form factors, keeping the full dependence on the heavy-quark mass. We presented the numerical evaluation of the form factors as well as their threshold limit, interesting for the calculation, in this kinematical region, of the total cross section for the photo-production of heavy quarks. This result is in complete agreement with the existing result in the literature.

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